

Lesson 12: Roofs

Roofs

An activity based on creating shapes on a grid with numbers, following given rules. This helps children to formulate ideas, talk about them and test patterns in shapes and numbers.

This activity has two episodes and leaves time for reflection.

Episode 1: From rules to spatial relationships and generalised number

Children discuss the isometric (triangular) grid and the shapes that can be made on it with four straight lines. They look at examples of roof-like and non roof-like shapes, and attempt to formulate the rules for them.

In the sharing phase, children sift through the rules generated, recognise similar rules, partial rules and those that are only true for some cases. They find the fewest rules needed and the simplest way of expressing them.

The agreed rules should now be used and checked, then refined. You can choose two of the numbers freely, but the other two will be tied to the rules. Use of symbols and algebra are discussed. More able children could tackle questions such as 'Why would that always work?' and 'Which two values determine the shape?'

Episode 2: Use of the idea of a variable in an expression

This is a short episode that allows children to find the number of roof-shapes that can be drawn on a given base when only the fourth value in the pattern is known. The generalisation from this episode reinforces the idea of 'any number' and the expression of the relationship in everyday language and, if appropriate, in letters.

Reflection: Recognising the value of generalisation and abstraction

Children should be able to speak about the processes they engaged in: following instructions, generating examples, generalising rules or relationships in words, sifting ideas, and more formal abstraction of the mathematical relationships between the four numerical values in the code for constructing a shape.

BEFORE YOU TEACH

To familiarise a class that has very little experience of working with an isometric grid, ask the children to draw as many different straight four-sided shapes on the grid as possible. Discuss similarities and differences and encourage the children to recognise that there are only two shapes possible: the parallelogram (and the rhombus, a special case of the parallelogram) and the trapezium. Ask the children to decide on the length of the side of their shapes, counting the distance from dot to dot, not the dots.

Roofs: Children's Thinking and Abstract of the Activity

3A
(Early Formal)

Episode 1c: Reaching the two-part rule

- Following each other's formulations as listed on board
- Sifting: recognising sameness of meaning and erroneous construction based on special cases only
- Recognising the need for two rules and selecting the best – each part is needed but is not enough on its own
- Formalisation moving to letters and symbols to represent any number – 1st and 3rd same, 1st and 2nd (or 2nd and 3rd) make 4th, $a, b, a, a + b$

Episode 2b: Expressing the relationship

- Converting from 'one less than the number' to a general number expression
- Recognising that the expression is itself a variable that is dependent on the value of the symbol(s) it contains

2B*
(Concrete Generalisation)
(reflection on operation; including classification/ordering etc.)

Episode 2a: How many roofs?

- Finding all roofs on a given base
- Generalising



'1 less than the base'
'always take away 1'
'chop off the top'

2B
(Mature Concrete)

Episode 1b: Drawing examples

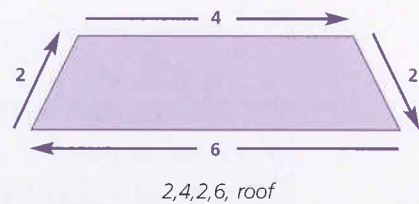
- Generating several examples from given sets of numbers
- Finding patterns and generalisations from examples and non-examples
- Sharing ideas

'first and third numbers are the same'
'the last number is the biggest'

2A/2B
(Middle Concrete)

Episode 1a: Preparation

- Looking at isometric paper and discussing which shapes can be made with four straight sides
- Following rules of matching a set of four whole numbers to sides of a roof; starting in the bottom left corner and going clockwise. Repeating for a 'non-example', such as 4,2,1,2



Piagetian levels

6

5

4

3

NC levels

Roofs

CAME Aims

- To develop some generalised 'rules' and relationships, including the finding and expressing of a two-part rule.
- Recognising the importance of examples and non-examples in reaching a generalisation.

Resources:

- Isometric dotted paper
- Dotty grid on board or OHT
- Scissors
- Roofs notesheet – one each

Organisation:

- Near ability pairs in mixed ability groups of 4 to 6

National Curriculum Reference:

- Algebra
- 2D shapes
- Number patterns
- Reasoning and justification

Vocabulary:

equal spacing,
straight sides, clockwise,
closed shape,
trapezium (roof),
rhombus (diamond),
parallelogram

1

Whole Class Preparation: (about 15 mins)

- Distribute the isometric dotted paper and the notesheet. If the children have done a pre-lesson, you could omit the the next four bullet points of this section or spend less time on them.
- *What do you notice about this paper?* Triangles, equal spacing.
- *Make as many different, four-sided shapes as you can.* Ensure the rows of dots on the paper are horizontal. Lines must be drawn to the nearest dot in any direction.
- Discuss similarities and differences. Some shapes will be the same but differently orientated. Some will be larger or stretched versions of others.
- Focus on the roof shape and talk about the significance of the lengths of sides.
- *We can draw a roof from a set of four numbers; some sets of numbers make a roof and some do not.* Give children some examples, including the two already drawn on the notesheet. Emphasise that the numbers refer to the length of the line, rather than the number of dots.
- Ask the children to use the convention of starting at the bottom left corner, then drawing up to the right, across the top to the right, angling down to the bottom right, and finally across the bottom to the left.

2

Group Work: (about 5–10 mins)

- Give out the Roofs notesheet.
- *In pairs, find out which sets of four numbers make a roof and which do not.*
- *Write down any patterns that you notice and then use this information to create a rule. If necessary, change the rule as you work.*
- Ensure that children are drawing roofs correctly, starting in the bottom left corner and working clockwise.
- Introduce counter examples to children who have discovered part of the rule (such as first and third numbers the same: 1, 6, 1, 1).
- Identify children who have noticed rules at different levels to contribute later.

4

Paired Work: (about 10 mins)

- *Supposing we have only the length of the base of a roof. How many roofs could we draw on top?* Model an example on the board or OHT for a base of 4. There are three possible roofs (tell them that the triangle is not allowed).
- *Try with another base length, such as a base of 5 or 8 or your own choice.*
- *Write down anything you notice.*

3

Class Sharing: (about 15 mins)

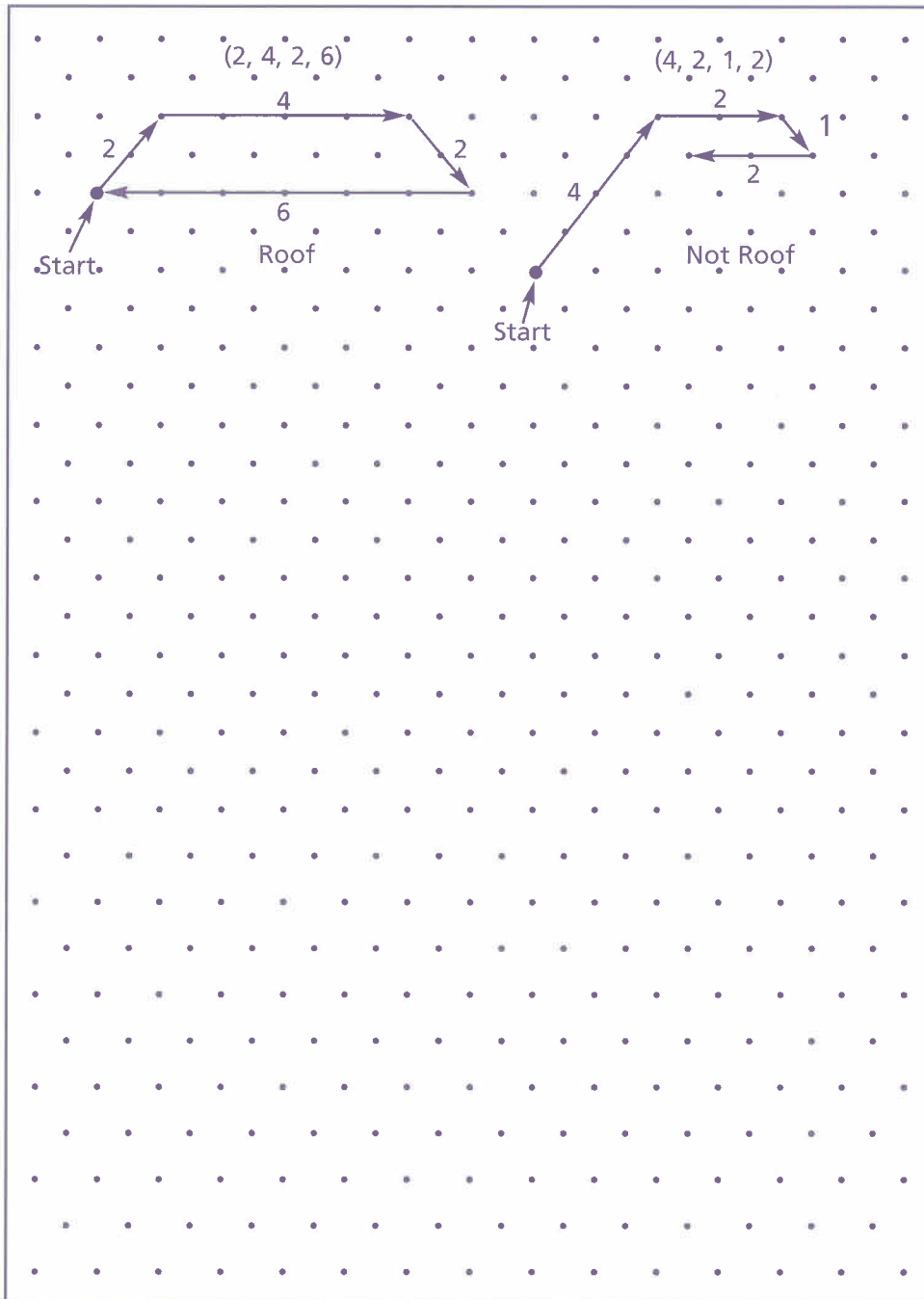
- Ask the children you identified earlier to contribute their findings, beginning with the simplest ideas. List rules on the board, valuing all patterns involving odd and even numbers, last number always the biggest, and so on.
- Go through rules one by one – *Are these true? Does this always produce a roof? Are there any 'not roofs' produced by this rule?*
- Discuss rules that are similar and how they are similar.
- Agree that some rules only apply in a few cases but not all.
- Identify the correct two-part rule (1st and 3rd, or 2nd and 3rd, are the same, 1st + 2nd gives the 4th).
- Make up sets of numbers with the children – *What could we put in the 1st place? What about the 4th place? Try some big numbers. Let's start at the end.*
- *What if we used n (any number) and m , or $*$ and \square ? Last gives $*, \square, *, * + \square$.*

5

Class Sharing: (about 5 mins)

- Choose several children to share examples and rules.
- *What if the length of the base was a big number, such as 100 or 350?*
- *What if the length of the base was n ?*
- *What did you do first in this lesson and how did you proceed?* Constructed examples. *What did you do with the examples?* Looked for rules and then expressed the rules in words or symbols.

Roofs



Draw roofs with these numbers: 2, 3, 2, 5 3, 1, 3, 4 3, 3, 3, 5
 1, 2, 1, 3 3, 2, 1, 4

Make a note of your ideas or rules.