Level	Stages in the development of thinking: multiplicative reasoning (ratio, proportion, scale)
Formal operational thinking: 3 and above	This level sees children being able to use ratio as a comparative tool to explore and model relationship mathematically. At the highest level of thinking children are able to use proportional reasoning to develop general laws to apply and use ratio to describe relationships between many variables. For example they can search information given involving 3 variables or more, and provide a solution e.g. given the change in bearings from a harbour to a buoy in reference to a lighthouse on the mainland they can find the distance of the buoy to the lighthouse. It is exemplified by the ability to think in the following ways or contexts: • at the highest level children now move towards an understanding of equilibria systems such as P1V1 = P2V2, • an understanding of scaling as a mental operation and they can apply it to concrete situations i.e. The wheel on Jay's wheelchair has a diameter of 52 cm and she moves forward 950 cm. How many times did the wheel rotate? ***When a car engine is turning at a rate of 2000 rpm the car is moving at 50mph. What would be the speed of the engine when the car travels at 80mph?*** • an understand of the difference between linear and quadratic functions as expressed graphically, • the ability to see ratios in relationships involving compound variables such as density as a weight/volume relation or work as a force x distance product or tangent as opposite/adjacent product etc. • the ability to compare ratios as proportions to look for equivalence (coloured paint, concentration of drink, sweetness of jam), • the ability to use inverse reasoning to see if a given ratio applies in a new context i.e. comparing leg to height ratio in a human with a toy doll and, • an understanding of functional relationships beyond linear i.e. moving from a fixed to a changing ratio (acceleration that is a further relationship building upon speed as a function of distance and time) as expressed graphically.
2B*	This level includes the ability to apply ratio beyond multiplication and division to understand the comparative nature of ratio as expressed in scale and proportion. It is exemplified by the ability to think in the following examples or contexts: • the ability to use ratio to scale up and down using appropriate numbers 3 x bigger = 1:3 and 3 x smaller = 3:1, • scaling now involves the use of percentage and equivalent fractions, • the knowledge that 'to scale' means all reduced/enlarged by the same amount so that the relative sizes remain constant, • understands the application of ratio in context of proportions (ie we are 3 X bigger than our heads) and so to calculate new values and quantities based upon this ratio, • beginning to apply proportional reasoning to other compound variable contexts: density, concentration, pressure, • the ability to see fractions as ratios i.e. 1/3 of 1/4 of is 1/12, ½ as twice as large as ¼ etc, • the understanding of ratio as a constant relationship between two values independent of size. This applies to a developing

	understanding of ratio as the gradient in graphs where m is the fixed multiplier (y = mX), • the ability to answer ratio questions involving whole number factor increase and halves. Children at this level are beginning to see ratio visually as proportion say in sloping rods. The rods getting further apart but size of gap is constant because the relationship between the two ratios, or gradients, is fixed.
2B	Here thinking includes the ability to deal with fractions as parts of a whole i.e. a ratio. As their thinking matures children develop the ability to deal with ratios in the following ways: • they are able to use % to think about fractions, e.g. 'a quarter is 25 %' plus an increasing confidence to deal with equivalence i.e. 0.5 = 50% = ½, • begin to link ratio to fractions i.e. '1:1 means 1 for you and 1 for me, that's 1/2, we both get ½'. • work with simple fractions and scaling that involves whole numbers including the ability to work out ratio via division, • be able to recognise doubling and tripling relationships as a general pattern and can formulate a rule: If I double this …… etc. At the lowest level thinking is limited to the ability to only process one relationship at a time with the notion of reversibility.
2A/B	This stage is typified by simple and comparative thinking which is exemplified in the ability to: • work out half and quarter divisions, • describe relationships between numbers, using ratio as a simple comparisons i.e. twice as much, • recognise and use multiples of 2, 5 and 10, • complete a series or sequence of multiples given a starting point: 3, 6 ……., 3, 15 ……. etc. At the lowest level children are still making the transition from repeated addition to multiplication i.e. "5 lots of 4 sweets – that's 20".