Primary CAME Thinking Maths

The lessons for the Primary Cognitive Acceleration in Mathematics Education (CAME) project in Year 5 and Year 6

Teachers’ Guide

Leverhulme Numeracy Research Programme (LNRP)
Focus Project 5: CAME in Primary School Mathematics

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The Primary CAME Thinking Maths materials evolved from focus project 5 of the Leverhulme Numeracy Research Programme. Focus project 5 lasted three years and is linked to ongoing research into cognitive acceleration.

Primary CAME Thinking Maths is published by BEAM Education.

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Published by BEAM Education
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London N1 3JT
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www.beam.co.uk
© BEAM Education 2002
Reprinted in 2006

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.
ISBN 1 903142 29 6
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Primary CAME Thinking Maths
Teachers’ Guide

“CAME theory has great benefits for class teaching. I think it allows class teachers to develop the teaching of mathematics, because of the extra thinking that it promotes in the lessons.”

A primary teacher working with CAME

Introduction

The introduction contains five short sections. The ideas contained within each of the sections will be elaborated on in the Primary CAME professional development programme.

Section one: A contribution to teaching
An explanation of how the Thinking Maths approach is situated within current practice in primary mathematics teaching.

Section two: The thinking behind Thinking Maths
A brief summary of the learning theories on which Thinking Maths is based. It is not essential to read this before teaching the first lesson. Reading it after the first two or three will help you make the most effective use of the lesson notes.

Section three: How to use Thinking Maths
Background information, including practical guidance and teaching style, to help you to begin using the Thinking Maths activities.

Section four: The format of the Thinking Maths activities
An explanation of the layout of all the components of the Thinking Maths lessons – background notes, teaching notes and notesheets, including the figure ‘Understanding lesson layout’.

Section five: The mathematical reasoning strands
A brief overview of the four main mathematical strands and the topics addressed in each of the Primary CAME Thinking Maths lessons.
Section one: A contribution to teaching

"CAME gives you a lot of freedom. There's not that pressure of thinking that the children have to know a certain thing by a certain date. It's more a case of the children learning what they can in the best way."

"We're talking about long-term benefits, not just covering programmes of study."

Primary teachers talking about CAME

The aim of the Primary Cognitive Acceleration in Mathematics Education (Primary CAME) project is to contribute to the teaching and learning of mathematics in Years 5 and 6. The Primary CAME Thinking Maths lessons are an outcome of the research from this project. They stimulate the development of children's mathematical thinking through carefully selected classroom tasks. Tackling these challenges encourages children to work together as mathematicians, constructing and discussing mathematical ideas.

Each Thinking Maths lesson promotes very specific mathematical connections and generalisations. However, children grapple with the 'big ideas' in mathematics, rather than focusing on the mastery of specific skills. The shared construction of mathematics encourages children to develop a deeper understanding of the mathematical concepts underlying the skills, algorithms and procedures in school mathematics. This includes those specified in the National Numeracy Strategy Framework for teaching mathematics from Reception to Year 6 (NNS Framework).

Primary CAME Thinking Maths lessons are not in themselves a mathematics scheme of work. Children need the regular content-based primary mathematics experiences of good instructional and open-ended investigational lessons. Thinking Maths lessons should be, at most, a fortnightly supplement to the normal mathematics experiences offered to children. The CAME approach complements and builds upon existing good practice in primary mathematics. It has been shown that Thinking Maths lessons, delivered in conjunction with good mathematical instruction and investigation, can significantly raise the thinking capacity of each child, as well as contribute to the meaningful learning of mathematics.

What makes Primary CAME different?

"In Thinking Maths lessons you end up with lots of questions."

"I agree. You can't stop thinking about them."

Two children talking after a Primary CAME Thinking Maths lesson

As mentioned above, the Primary CAME approach shares many of the features of existing good practice in primary mathematics: children are encouraged to talk, listen, question and debate their mathematical ideas.

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CAME is open-ended in terms of the conceptual points that children will reach but very focused in terms of what tasks the children are set. Primary CAME often addresses concepts thought of as beyond the scope of primary mathematics, sowing seeds for later mathematical work. At all times, the emphasis is on depth of mathematical thinking and conjecturing in which children construct mathematical ideas and gain insights at different levels of complexity, all within a mathematical ‘big idea’.

The lessons stimulate children’s mathematical thinking and often leave children with unanswered or partly answered questions. This mathematical ‘unfinished business’ will either be addressed in further Thinking Maths lessons or as part of the normal mathematical curriculum, or later in the children’s mathematical career. Thinking Maths lessons differ from good instruction and practice lessons in that each one has a clear agenda, involving fundamental concepts in mathematics. The lesson focuses on children ‘struggling on the way’ towards these ideas, rather than on fully understanding and mastering the concepts. The outcomes of a Thinking Maths lesson are the thinking processes and the sharing of ideas rather than the specific knowledge gained and skills employed.

Hence, although children are working in an investigative way throughout these lessons, Thinking Maths lessons do differ from many good, open-ended investigations or problem-solving lessons. The CAME activity is very focused in terms of the tasks that children tackle, but open-ended in the mathematical understandings that children reach. The lessons provide clear challenge points rather than allowing varied interpretations of the task.

Whole-class teaching

“CAME lessons are an opportunity to develop children’s self-esteem with number, because you value every contribution.”

A primary teacher talking about CAME

Thinking Maths lessons are centred around whole-class activities. The whole class tackles the same challenges together and in small groups. A key feature is the sharing and discussion of children’s mathematical constructions as a class. Classes involved in the development of Primary CAME Thinking Maths lesson materials have successfully included children of a wide range of abilities through careful grouping and support. Different children reach different levels of thinking. So, rather than differentiation by task, differentiation is by thinking outcome within a task.

CAME, the National Curriculum for Mathematics and the National Numeracy Strategy Framework for teaching mathematics from Reception to Year 6

“More time in mathematics [should be] devoted to interaction between teachers and pupils about mathematics, especially in interactions with the whole class, and in groups.”

Desirable Outcome from Numeracy Matters: the preliminary report of the Numeracy Task Force; p.53
The CAME approach fits well within the NNS Framework and the requirements of the National Curriculum. Thinking Maths lessons delivered as part of a balance of mathematical activities are consistent with the requirements of the NNS Framework. Each Thinking Maths lesson outline contains rough indicators of how each part of the activity fits in with National Curriculum levels. However, Thinking Maths aims to promote children’s understanding of the underlying mathematical ideas rather than to teach specific skills or procedures as highlighted in the National Curriculum or the NNS Framework. Thus, it is important to emphasise here that the National Curriculum indicators do not give a full flavour of the mathematical thinking within a lesson.

The lessons have been developed primarily for children working at National Curriculum level 2 and above. This is the performance achieved by the average seven-year-old. Level 4 is the performance generally achieved by the average eleven-year-old. However, in the development classes, children working below this level were able, with support, to successfully participate in the lessons.

The Primary CAME programme functions as a natural introduction to CAME in the secondary school, a programme of 30 lessons for Year 7 and Year 8. Indeed, one of the aims of the programme is to prepare children for secondary CAME and to maximise the impact CAME lessons have on pupils at Key Stage 3 and beyond. However, Primary CAME is also ideal for use as a stand-alone programme in Local Educational Authorities that have not implemented CAME or CASE (a parallel programme in science) in their secondary schools.

More than 35 lessons were considered for inclusion in the Primary CAME programme; the 24 activities chosen were selected for their effectiveness during trialling. Most of the activities in the Primary CAME programme were devised during the Leverhulme Numeracy Research Programme, the rest are Secondary CAME activities that have been refined and adapted for use with children in Year 5 and Year 6.

Although the CAME activities have been adapted for use with a different age group, they retain the context used in their original form. For those children who study both Primary CAME and secondary CAME activities, the overlap between some of the activities from the two programmes will allow children to progress even further in their mathematical thinking as less time is required for familiarisation with the context.
Section two: The thinking behind Thinking Maths

"We are looking to move children's thinking forward, and not all children are going to get to the same point at the end. That's not what it's about. We are trying to raise awareness, so that the children will continue to think about the problem even when the session is over."

"That's what's nice. You can leave it in the air. It's the talk."

Primary teachers talking about CAME

Mathematics is a shared activity and is one way in which children seek to make sense of the world. If children are to become successful, mature thinkers and learners, they need to develop ever more general and abstract mathematical frameworks incorporating mathematics' 'big ideas'. However, simply telling children about 'big ideas' does not work. Children's need to understand often leads them to develop misconceptions and misunderstandings of key mathematical concepts and procedures, which they will need to address.

The CAME approach to teaching makes use of children's thought processes to enable them to develop greater mathematical understandings. To do this, CAME draws upon two theoretical approaches to children's learning: Piagetian theories about the individual development of mental powers and Vygotskian and social constructivist theories about the social interaction that drives this individual development.

The Piagetian contribution

Piaget described children's thinking in terms of qualitatively distinct stages. The broad stages that concern us in Years 5 and 6 are concrete operational thinking and, to a lesser extent, formal operational thinking, where a child has begun to think through logical propositions.

Concrete operations are thought processes based on a person's perceptions. The images and ideas to be worked on may arise from practical activities or from something read, seen, heard or even imagined. The crucial characteristic of concrete operational thinking is that children have the ability to describe situations but not to explain them beyond simple cause and effect. By contrast, formal operational thinking allows people to step back from a problem and to create or use general explanatory models of events and to test how accurate those models are.

Piagetian ideas are particularly powerful in describing the underlying difficulty of mathematical learning tasks in the same terms as those used to describe children's thinking. Piaget's thinking stages are very broad. Subsequent research has divided these into sub-stages or thinking levels. Concrete operational thinking is sub-divided into early, middle, mature and concrete generalisation. Formal operational thinking is sub-divided into early formal and mature formal. These new sub-stages provide an enriched description of children's thinking in a Thinking Maths lesson.
Formal operational thinking allows people to generate an idea (model) about events already described through concrete operations, and then to test how well the idea connects the events.

For example, if a simple graph was plotted of the height of a growing plant against the number of the weeks it has been measured for (Figure 1), a child using concrete operations can readily answer questions such as: ‘Between which weeks did the plant grow fastest?’ This is because there is a direct relation between the steepness of the graph and the rate of plant growth. The one describes the other.

Suppose the task is to examine the three graphs in Figure 2 and then to answer questions such as: ‘Which of these best describes a lift rising from the ground to the fifth floor and stopping twice on the way up?’ or ‘Point to a part of a graph that represents an impossible journey’. A child using descriptive models only (concrete operations) is apt to choose a or b as the answer to the first question and to offer no answer to the second. The ability required is to step back from the problem, and to say to oneself: ‘This is about the relationship between the height axis and the time axis’, then ‘If the graph is going vertically, this would mean the lift is travelling up in zero time’ which is impossible, and finally, ‘The graph with the lift movement shown at an angle must represent its travel in time’. To do this requires formal operational thinking.

All but the most exceptionally able Year 5 and Year 6 children will be concrete operational thinkers. A few children may still be pre-operational thinkers – able to recognise and name things but unable to think through the results of actions without actually carrying them out. Thinking Maths lessons aim to help children become more mature and more successful concrete operational thinkers across the range of mathematical ‘big ideas’, preparing the way for their development as formal operational thinkers. The potential of this approach is demonstrated by the significant
results that have been achieved in the Leverhulme Numeracy Research Programme Focus 5 Primary CAME project research, particularly in terms of Piagetian measures. Significant results have also been demonstrated through Piagetian and Mathematical Reasoning achievement test results in Year 8 at the secondary level and in national tests at the end of Key Stage 3.

The hypothesised Piagetian level of thinking associated with the different conceptions and understandings within the lesson is shown on the left hand side of each Thinking Maths lesson abstract. As with National Curriculum levels, these are only indicators of thinking difficulty. They are not intended to give a full picture of every child’s thinking during the lesson.

**Vygotsky and children's development**

Mathematics is a social activity. Mathematical concepts have been developed over time by people working together. Similarly, children construct and discuss mathematical understandings together. It is through participation in the mathematics classroom and discussion of mathematical meanings that children learn to become mathematical thinkers.

In this respect, Piaget's description of development as an interaction between an individual and their environment is inadequate on its own. Those involved in the development of Primary CAME see Vygotskian theories of learning as providing the necessary social complement to Piaget's ideas.

Vygotsky emphasised that children's learning is mediated by other people. In early childhood, the child's immediate world is framed, simplified and thus mediated by older family members and adults in general. However, as children become older and engage with tasks in and outside school, their main mediators are their peers. Children learn through interacting with and listening to their peers. Although they still do some of the work in developing their thinking for themselves, more usually, they see or hear another pupil showing an idea which is just beyond the competence level they are at. They then immediately make that idea their own.

The CAME approach uses Vygotsky's work on development and Piaget's work on children's thinking to put children in a position where, in collaboration with their peers, they must construct key mathematical reasoning patterns for themselves.
Section three: How to use Thinking Maths

“CAME is all about allowing children to work out what is going on for themselves but making sure that at the end of a session you, the teacher, bring all those ideas together again, guiding but saying as little as possible yourself.”

A primary teacher

Thinking Maths lessons are designed to help children develop the general thinking skills that are required in mathematics. Children are first asked to describe the task they are presented with in their everyday language, and to suggest strategies to carry it out. Then they are asked to work together in small groups, with the goal of presenting their solutions or problems to the rest of the class. In whole-class discussion, individuals and groups are invited to contribute their ideas and, finally, the class is asked to summarise and reflect upon each of the important points.

The teacher’s role

“As much as possible, the teacher’s role is just to guide the discussion rather than to lead it and say what the magic formula is.”

“You’re making yourself vulnerable in a sense, because you’re not asking closed questions and you don’t necessarily know what you’re going to get from the children.”

Primary teachers talking about CAME

The role of the teacher in Thinking Maths lessons is threefold. First, the teacher must be able to look ahead on behalf of the child. The mathematical and thinking aims within a conceptual strand are long term and the teacher frames the specific challenges so that children develop in the right direction. Of course, children can and do constantly surprise us with their inventive and imaginative mathematical constructions. Mathematics teaching is a learning experience for the teacher as well as the children.

Second, the teacher is a manager of children’s discussion. Those involved in the development of Primary CAME believe that much of the development of children’s thinking comes from internalising, in a flash, some better understanding they see or hear from another child who is just beyond the level they have reached. Each Thinking Maths lesson is structured to produce as many and as wide a variety of these fresh insights and competencies as possible. As well as reporting their own thinking, children must listen and respond to the reports of others’ thinking. Ideally, the teacher steps back from the discussion when children are confident enough to question and discuss their constructions directly with one another. Thus, this role goes beyond good questioning to promoting good listening, questioning and dialogue by and between children.

Third, the teacher needs to encourage a classroom culture in which enquiry, collaborative learning and the sharing of ideas become dominant themes. They should discourage mathematics learning being viewed as just an individual activity where children expect to
be trained in the application of formal rules and procedures. The development of this kind of classroom culture is facilitated through the creation of an environment in which the contribution of each child is valued and taken as valid for some purpose. The aim is to develop each child’s mathematical thinking to its fullest potential rather than simply to achieve ‘correct’ answers.

**Lesson structure**

Each lesson activity is centred around one or two mathematical ‘big ideas’, although the intention is for children to ‘struggle’ on the way towards these ideas rather than be taught specific knowledge or skills. Lessons last from 60 to 90 minutes.

Every lesson begins with an introduction, where both the language and context chosen should be within the grasp of the whole class, that is, it should be a familiar context that requires only early concrete operational thinking. An example might be recognition and single-step mental operations on familiar objects that may or may not be mathematical. The main purpose of this introductory activity, which is called the preparation phase, is to ensure that all children have sufficient grounding in the context, and confidence in the use of any technical terms introduced at this point, for the work that follows.

The preparation phase is followed by an activity that requires a higher level of thinking (mature concrete in most cases). The children know what the problem is about, know what the words mean, but need to shift their thinking to a higher level in order to cope with the new problem. This is typically done in small groups. This phase is known as the construction phase, and it is here that the children’s informal strategies or ideas often lead them to see some contradiction or cognitive conflict. This is followed by the children coming together in whole-class discussion where they share their constructions and reflect upon their thinking.

A cycle of the three phases – preparation, construction and sharing – is referred to as an episode and a typical episode consists of some introductory work with the whole class, followed by small group work and then returning to whole-class discussion to share ideas.

Each Thinking Maths lesson consists of two or three episodes, with each episode designed to provide challenges at a higher level of cognitive demand than its predecessor. The last phase of the first episode generally provides the preparation for the second episode, and so on. The last episode should also include time for reflection.

Different classes will reach different points in a given Thinking Maths lesson. The aim is to promote children’s thinking rather than to reach a particular common end point. The teacher should, however, ensure that at least one episode is completed and time reserved for a last reflection phase – this is crucial and needs to be included, regardless of how far the class has progressed.

Further descriptions of the three phases in each episode are given on the following page.
The preparation phase

Most activities start with a problem set well within the children’s experience, which we call concrete preparation. The understanding barrier is set deliberately low for this initial phase, which often involves invoking links to previous ideas or previous Thinking Maths lessons. During this phase, the teacher needs to get the children involved in the problem by asking questions, sharing diagrams on the board, and so on.

The construction phase (and cognitive conflict)

Construction is the process by which children generate more powerful strategies or concepts. In Thinking Maths lessons, some construction may take place in the concrete preparation phase or later, during whole-class discussion, but most takes place when groups of children collaborate on tasks and together develop ideas to solve a problem.

Thinking Maths lessons often lead children to produce solutions that contradict their existing ideas. For example, in Lesson 12: ‘Roofs’, children may produce what appears to be a perfectly good rule for making a ‘roof’, only to find it disproved by a non-confirming example. This sort of surprise, or cognitive conflict, is an important element in cognitive development, promoting further and more sophisticated construction – it challenges children to produce a higher-level strategy that does work.

Features inherent in school-level mathematics provoke cognitive conflict. Much of school mathematics is based on the idea of consistency, and recognising an inconsistency provides an opportunity for provoking higher-level thinking. These are the opportunities that Thinking Maths capitalises on.

The sharing and reflection phase (and bridging)

During this phase of the lesson, children reflect on their work and share the ideas they have constructed. Reflecting on one’s own actions or one’s own thinking is referred to as metacognition. It is important to encourage this reflection: without it children may easily lose the gains that they have made. In the Thinking Maths model, metacognition is primarily accommodated for in the class sharing of the many insights, difficulties and partially or completely successful strategies that have occurred in the construction phase(s) of the lesson. Metacognition leads naturally to, and is often helped by, the process of bridging.

Bridging is usually part of the reflection phase in the final class discussion, and normally takes place near the end of the discussion. The teacher asks the class to collaborate in finding ‘good names’ or phrases to describe what has been achieved and why or how it worked, and then records them on the board in the children’s own words. The children are then asked to suggest other contexts where the reasoning pattern or the mathematical relation that underlies the Thinking Maths lesson might be applied. This leaves a ‘handle’ in children’s long-term memory with which the concept can be retrieved on another occasion, and applied to a new context.
Implementing the Thinking Maths approach

Learning mathematics is a shared activity for adults as well as children. The more it is treated as an individual activity, the more difficult mathematics becomes. This makes it difficult to grasp the CAME approach simply by reading the pages of a teachers’ guide. For this reason, the Primary CAME Thinking Maths Teachers’ Guide is only one part of the Primary CAME professional development programme.

In the Primary CAME professional development programme, the teacher’s first experience of a lesson is in a simulation, during which they engage with the children’s possible constructions in the context of the mathematical ‘big ideas’.

They then begin to teach the lessons. After delivering the lessons, teachers reflect on the experience in the context of CAME’s theoretical basis for teaching and learning: increasing levels of cognitive demand and encouraging children to work together and share ideas.

Teachers have found it helpful to observe others teaching, to be observed by colleagues, and to team-teach lessons. They will often teach a lesson a second time to an older, younger or parallel group of children in order to better understand the lesson ‘flow’ and potential variations in differentiation between individuals and groups.

In summary, the Primary CAME programme includes the delivery of the Thinking Maths lessons, teachers observing and team-teaching Thinking Maths with other teachers, and teachers talking and reflecting upon the lessons, together with the more formal Primary CAME professional development sessions. Each of these aspects contributes to accelerating the learning of all those involved in the programme – children and teachers alike.
Section four: The format of the Thinking Maths activities

Each of the Thinking Maths lessons is outlined on two double-page spreads, which are followed by any notesheets that you may need for teaching the lesson.

The double-page spreads – Background Notes and Teaching Notes – are explained in this section. An explanatory diagram ‘Understanding lesson layout’ is on pages 20 and 21. Remove those pages from the folder now so that you can refer to them as you read through this section. You may also wish to take out pages 36 to 39, the full-size version of the lesson annotated on pages 20 and 21.

Before teaching a lesson, you will need to read the Teaching Notes spread as well as the ‘Before you teach’ section from the first page of the Background Notes. You may find it helpful to add your own notes to these pages, either before or after you have taught a lesson.

The Background Notes double-page spread (the narrative overview of the lesson and the diagram that provides information on ‘Children’s Thinking and Abstract of the Activity’) will probably be something you return to over time. The information in them is generally more helpful after you have taught the lesson or set of lessons from the same mathematical reasoning strand (see Section five).

The notesheets are for children’s rough workings. These should not be collected or marked. Teachers often just ask the children to make notes on scrap paper, using the notesheets as prompts.

The Background Notes pages
The Background Notes form the first two pages of each lesson. They are not intended to stand alone but should be viewed together with the Teaching Notes and notesheets (if the lesson includes any), to get a more complete idea of the flow of ideas in the lesson.

These pages include a narrative description and a levels diagram, as shown in the example on page 20.

The narrative description
The first paragraph of each lesson provides a very brief sketch of the aim of the lesson, naming the ‘big ideas’ involved. In Lesson 3, we are told that the intention is to provide children with an opportunity to explore different methods for finding perimeter and area of rectangles. This is followed by a short description of the flow of the lesson.

The main description of lesson aims is presented in boxes, and is preceded by a reminder about the structure of each episode, each of which includes an introduction, paired or group work and whole-class sharing. As with most Primary CAME Thinking Maths lessons, Lesson 3 has two episodes. The first episode introduces ideas that are explored in greater depth in the second episode. The last box contains a brief note regarding the final reflection phase.

Immediately below the box is the section ‘Before you teach’ which tells you some of the key things to consider before teaching the lesson. It highlights some of the main
Section four: The format of the Thinking Maths activities

cconcerns that may arise, and what should be avoided (for example, don’t hurry the
initial phases to arrive at the formal mathematics). Here we also point to particular
strategies or misconceptions that need to be addressed (such as counting squares
rather than edges to find perimeter). This section also indicates any specific knowledge
or experience the children may need before a lesson. This will help ensure that
the main purpose and flow of the lesson is not disrupted on the day.

The levels diagram – Children’s Thinking and Abstract of the Activity
The diagram on the second page of the Background Notes is a flow diagram to be read
from the bottom up. It parallels the narrative on the previous facing page and gives an
idea of the cognitive demand of each stage of the activity, in terms of Piagetian
levels and National Curriculum levels. It should be noted that these levels are only for
teacher guidance. There is not meant to be a direct match between the cognitive
demands of a task that is concerned with reasoning, and a particular National
Curriculum level linked with content or topic. The levels shown provide an indication of
what type of thinking is being promoted. The narrative portion of the diagram provides
further indication of what is taking place in each of the episodes. In the
‘Largest Rectangle’ the lesson begins with tasks at a lower level of cognitive demand
and of National Curriculum levels. The children talk about rules in the final sharing
phase of the first episode.

The rules then become the means for moving on to the second episode, the different
stages of which are also roughly equated to Piagetian and National Curriculum levels. This
episode is more demanding, particularly in terms of cognition (concrete generalisation
moving to early formal).

The Teaching Notes pages
The Teaching Notes pages are two facing pages that include both general information
and specific guidance for teaching the lesson. See the example on page 21.

General preparation
The first page includes information to help prepare for the lesson:

• CAME aims – a brief indication of the lesson’s main thinking points and mathematical
  content.

• Resources – a list of the special resources needed. ‘Largest Rectangle’, for example, asks
  for special paper and the rectangles notesheets.

• Organisation – which size and ability range of groups have worked well during the
  trialling of each lesson.

• Vocabulary – key mathematical vocabulary for the lesson. Note: this is not a
  key focus in the lesson and the ideas are generally introduced in the context of
  children’s everyday language.

• National Curriculum references – an indication of broad National Curriculum
  strands which the children will be working within during the lesson. Note: they are
  listed here to help link the ideas from a Thinking Maths lesson to other lessons, not
  to indicate what is to be taught.
Lesson delivery
The bottom of the first page of the two-page teaching spread provides material for introducing the lesson activity to the whole class, denoted as ‘Whole Class Preparation’ in box 1. Suggestions for opening questions, such as, ‘Is a square a rectangle?’ are given in italics. Other general suggestions for conducting this preparation phase are given in roman type face.

The second page of the teaching spread provides background and suggestions for questions for continuing with the first episode. ‘Paired Work’ or ‘Group Work’ (box 2) is the first construction phase, during which children work on the initial stages of a task together. This is followed by the ‘Class Sharing’ phase (box 3). During this phase, the class talks about what they have discovered. The first episode ends with the children sharing the rules they have found (in the case of Lesson 3, using words and, if suggested by the children, letters for length and width). Notice that the boxes for the first episode are shown in the foreground and are numbered 1, 2 and 3.

The lesson carries on with the second episode – the shaded boxes numbered 4 and 5. Box 4 sets out paired work or further constructions, while box 5 returns to class sharing. Note: depending on the time available, you may need to bring things to a close well before getting into the later work. If you do, the final questions should be adjusted to ask the children to reflect on where they have reached and what they have found, as well as how have they have addressed the initial questions.

The notesheets
Some lessons use two or three notesheets while others use none. The notesheets are there to support the delivery of the lesson. Remember that these are provided for children’s rough workings and should not be collected or marked.
The diagram breaks down the lesson into what children will be discovering at each level of cognitive demand.

*Both Piagetian and NC levels on the diagram only provide a general indication of level of demand. Note that this level may not directly correspond with the text box description.*

The material to the right-hand side of the boxes provides teachers with hints and examples to use at each stage of the lesson.
Section four: Understanding lesson layout

The Teaching Notes pages start with information to help you prepare the lesson, including a summary of the lesson's aims, resources that you may need, vocabulary that may be used, organisation of the class and an indication of links with the National Curriculum.

Boxes 4 and 5 that deal with Episode 2 are set back from the page to demonstrate that they represent the later stages of the lesson. Remember that whatever stage you get to, you must have a reflection phase at the end of the session.

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Primary CAME Thinking Maths

Largest Rectangle

**CAME Aims:**
- Ordering a range of rectangles by estimating their area and perimeter.
- Reflecting on different methods of finding area and perimeter.

**Resources:**
- Transparent square overlays
- Overhead projector
- Ruler
- Whiteboard
- Worksheet
- Number cards

**Vocabulary:**
- square, rectangle, width, height, length, area, perimeter

**Organisations:**
- Whole class together on the board.
- Pairs, working on the board.

**National Curriculum References:**
- Properties of rectangles and squares
- Area and perimeter
- Addition and subtraction

---

Primary CAME Thinking Maths

Paired Work: About 10 mins

- Pair with a partner and try to find more than one pair of rectangles that have the same area.
- How many pairs can you find?
- Discuss your findings in pairs.

Paired Work: About 10-15 mins

- How do you know that the rectangle has a smaller area?
- How do you know that the rectangle has a smaller perimeter?

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Primary CAME Thinking Maths

Paired Work: About 10 mins

- Pair with a partner and try to find more than one pair of rectangles that have the same area.
- How many pairs can you find?
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Paired Work: About 10-15 mins

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Paired Work: About 10-15 mins

- How do you know that the rectangle has a smaller area?
- How do you know that the rectangle has a smaller perimeter?

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Whole Class Preparation (About 10-15 mins)

- Describe a rectangle. When you are creating a square, what is the best way to make a square that is a good size?
- Which of the rectangles is the largest?
- What is the difference between a square and a rectangle?
- What units are they measured in?

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This box and those following it contain material to help teachers deliver the lesson. Box 1 demonstrates how to introduce the lesson activity to the class. Box 2 provides suggestions for Episode 1’s construction phase, while box 3 contains ideas to help the class share what they have learnt. Boxes 4 and 5 are similar in content to 2 and 3 respectively but apply to Episode 2.

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Section five: The mathematical reasoning strands

The suggested order for teaching the lessons to a class of Year 5 or Year 6 children is given in the table below. Each lesson has a particular focus in terms of reasoning strands in the mathematics curriculum:

- number
- number relations and algebra
- shape and space
- data handling and measurement

The suggested order is based on balancing type of lesson and gradual increasing challenge in mathematical reasoning over the full set of lessons and the lessons within each strand.

When planning the mathematics work over a term, you may wish to incorporate a particular Primary CAME lesson into an appropriate unit of work, teaching the lesson ‘out of order’. Alternatively, each lesson could be taught ‘outside’ the medium term planning, in a time slot allocated for problem solving or assessment and review. The table includes both an indication of some curriculum links and the main strand for each lesson.

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<thead>
<tr>
<th>Year 5</th>
<th>Curriculum links</th>
<th>Focus</th>
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</thead>
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<td>Multiplication, algebra</td>
<td>Number relations</td>
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<tr>
<td>Digit Detective</td>
<td>Inverse operation, place value</td>
<td>Number</td>
</tr>
<tr>
<td>Largest Rectangle</td>
<td>Area, perimeter, properties of rectangles</td>
<td>Shape</td>
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<tr>
<td>Share an Apple</td>
<td>Fractions</td>
<td>Number</td>
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<tr>
<td>Scaffolding</td>
<td>Number patterns, ratio, algebra</td>
<td>Number relations</td>
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<tr>
<td>Comparing Texts</td>
<td>Mode, range, representing data</td>
<td>Data</td>
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<tr>
<td>Picturing Numbers</td>
<td>Mental calculation, vertical number line</td>
<td>Number</td>
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<td>Cups and Saucers</td>
<td>Circles (pre-pi), ratio</td>
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<td>Mini Clubs</td>
<td>Calculating, properties of numbers</td>
<td>Number</td>
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<tr>
<td>Tessellating Triangles</td>
<td>Translation, rotation, reflection, angles</td>
<td>Shape</td>
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<td>Design a Desk</td>
<td>Length, averages</td>
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<tr>
<td>Roofs</td>
<td>Number patterns, algebra</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Year 6</th>
<th>Curriculum links</th>
<th>Focus</th>
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<tr>
<td>Halving and Thirding</td>
<td>Fractions, ratio</td>
<td>Number</td>
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<tr>
<td>One-way Tracks</td>
<td>Number properties, networks</td>
<td>Shape</td>
</tr>
<tr>
<td>Beanbag Pick Up</td>
<td>Properties of numbers, algebra</td>
<td>Number relations</td>
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<tr>
<td>Which Way Up?</td>
<td>Probability, data, inverse relationship</td>
<td>Data</td>
</tr>
<tr>
<td>Pencils and Rulers</td>
<td>Calculation, two-way relationships</td>
<td>Number relations</td>
</tr>
</tbody>
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